Victoria Shanghai Academy

IB MYP YEAR 5

YEAR 10 Mathematics

Assessment #3
MATRICES

Name: ______________________________ (10)

Ans

Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, December 14, 2012** Time allowed: **95 mins**

Student's **Performance** in Different **Criterion**			
B		**D**	

INSTRUCTIONS:

- Read the instructions for all questions carefully.
- Show all work, steps and proper units.
- Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- Write in **PENCIL**.
- **NOT** allowed to use any **electronic devices**, such as translators.
- Allowed to use **GDC**.

ASSESSMENT:

- Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- This task assesses Criteria **B & D** considering ALL the questions.

Criterion B: INVESTIGATING PATTERNS **(ONLY APPLICABLE TO QUESTION IN PART 1)**

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	
1–2 **Do Maths**	The student performs appropriate calculations (in (a) to (c), in (g), to (i), and in (k) to (m)) in order to recognize simple patterns.	The student **applies, with some guidance**, mathematical problem-solving techniques to recognize **simple** patterns.	**(0-8)**
3–4 **General Rule**	The student correctly solves (d) and (e) and suggests general rules in parts (f) and (n).	The student ● **selects and applies** mathematical problem-solving techniques to recognize patterns, and ● **suggests** relationships or general rules.	**Teacher's Final Grade**
5–6 **Test it**	The student describes relationships (in (j) and (n)) mathematically, and connects the various relationships. If a student has drawn conclusions consistent with their findings in part 2 (g), credit ***may*** be given here.	The student ● **selects and applies** mathematical problem-solving techniques to recognize patterns, ● **describes** them as relationships or general rules, and ● **draws conclusions** consistent with findings.	4-8 **(0-8)**
7–8 **Prove it**	The student is able to correctly prove mathematically all the relationships and rules seen in the task, and is successful in (j) and (n).	The student ● **selects and applies** mathematical problem-solving techniques to recognize patterns, ● **describes** them as relationships or general rules, ● **draws conclusions** consistent with findings, and ● **provides justifications or proofs.**	

Criterion D: REFLECTION (ONLY APPLICABLE TO QUESTION IN **PART 2**).

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-6)
1–2	The student's answers to questions (a) and (b) describe the meaning of his/her findings in real life.	• The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. • The lines of reasoning are **difficult to follow**.	
3–4	The student's answers to questions (c) (i) to (iv) comment on how his / her findings make sense in real life situation.	• The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. • The lines of reasoning are **clear** though not always **logical** or **complete**. • The student moves between different forms of representation **with some success**.	Teacher's Final Grade (0-6)
5–6	In question (d), the student is able to recognize and explain the applications seen in the vector operations. In question (e), the student has explained how this answer makes sense in the context of this problem.	• The student shows **good** use of mathematical language **and** forms of mathematical representation. • The lines of reasoning are **concise**, **logical** and **complete**. • The student moves **effectively** between different forms of representation.	

A Special Matrix

Part 1

This section is assessed against criterion B only
You should spend 45-50 minutes on this section, and you are advised to make full use of your GDC.

Consider the following matrices:

$L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $M = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

(a) Find the matrix L^2

$$L^2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

(b) Find the matrix M^2

$$M^2 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$$

(c) Find the matrix N^2

$$N^2 = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$$

A student, Xavi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ then A^2 is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

(d) Find A^2

$$A^2 = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & 2a^2+2a+1 \end{pmatrix}$$

(e) Is Xavi right?

Yes, if $A^2 = \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

ie. $b = 2a^2+2a$

(f) If your answer to **(e)** is "yes", is Xavi ***always*** right? Explain.
If your answer to **(e)** is "no", why is he wrong? Explain.

Yes. Xavi is always right because the matrix product in part (d) will work for all values of a.

(g) Find the matrix LM

$$LM = \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}$$

(h) Find the matrix MN

$$MN = \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}$$

(i) Find the matrix LN

$$LN = \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}$$

Another student, Messi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ and B is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

then the matrix AB is always of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(j) Prove (or disprove) Messi's hypothesis

$$\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b+2ab+1 & a+b+2ab \\ a+b+2ab & a+b+2ab+1 \end{pmatrix}$$

$$= \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix} \text{ where } c = a+b+2ab$$

Now go back to the original matrices.

(k) Find L^3

$$L^3 = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$$

(l) Find L^4

$$L^4 = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$$

(m) Find L^5

$$L^5 = \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$$

(n) What do you notice about the form of the answers to (k), (l) and (m)? Try to use the various answers you have in order to generalize.

$A^n = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \Rightarrow A^{n+1} \begin{pmatrix} 3a+2 & 3a+1 \\ 3a+1 & 3a+2 \end{pmatrix}$.

where $b = a+1$

if. $A^n = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \Rightarrow A^{n+1} = \begin{pmatrix} 2a+b & a+2b \\ 2b+a & b+2a \end{pmatrix}$.

if $A^n = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \Rightarrow A^{n+1} = \begin{pmatrix} 3a-1 & 3b+1 \\ 3b+1 & 3a-1 \end{pmatrix}$.

(n) We notice that the answers to (k), (l) and (m) are also of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

This would lead to the hypothesis:

For matrices of the form $A = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$, A^n will be of the form $\begin{pmatrix} k+1 & k \\ k & k+1 \end{pmatrix}$

This actually follows from previous results:

(i) We have shown that, for matrices of the form $A = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ matrices of the form A^2 can be written $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

(ii) A^3 can be written as A^2 x A, and A is of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ while A^2 is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

(iii) We have shown that such a product - $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$ - will be of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(iv) This means that A^3 will be of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(v) A^4 can be written as A^3 x A, and we can iterate through these results again.

Thus A^n will be of the form $\begin{pmatrix} k+1 & k \\ k & k+1 \end{pmatrix}$

$L \quad \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$L^2 \quad \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

$L^3 \quad \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$

$L^4 \quad \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$

$L^5 \quad \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

Bad guy runs and cop chases... "*Catch me if you can!*"

Part 2

This section is assessed against criterion D only. You should spend 35-40 minutes on this section.

One day, a thief (T) stole a handbag from a lady (L) , the lady shouted for help, a cop (C) was nearby, he then ran after the thief.....

It is given that the position of T, C and L are (3, 14), (1, 8) and (k, -2k) respectively, where k is a constant.

(a) (i) Express $\vec{CT}$ in terms of **i** and **j**.

C (1, 8) . T (3, 14)

$\vec{CT} = (3-1)\vec{i} + (14-8)\vec{j}$

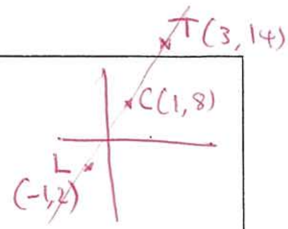
$= 2\vec{i} + 6\vec{j}$

(ii) Express $\vec{TL}$ in terms of **i**, **j** and **k**.

T (3, 14) . L (k, -2k)

$\vec{TL} = (k-3)\vec{i} + (-2k-14)\vec{j}$

(b) If the value of k is -1, describe the meaning of this situation in real life. Show your work briefly.



ie $L(-1, 2)$

$\therefore \overrightarrow{TL} = (-4\vec{i} - 12\vec{j})$

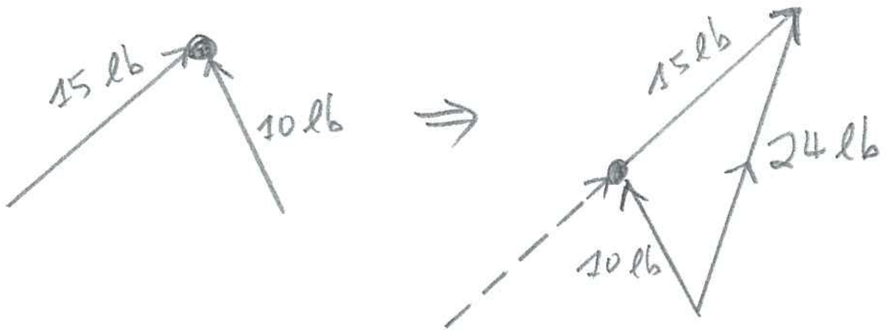
$= -2(2\vec{i} + 6\vec{j})$

$\because \overrightarrow{CT} = -2\overrightarrow{TL}$

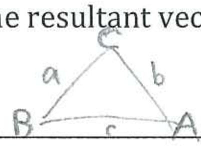
$\therefore$ T, C and L are collinear ie. They're on a straight line.

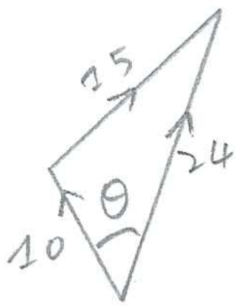
A few minutes later, T and C met… they started fighting for the handbag. Two forces with magnitudes of 15 lb and 10 lb were applied to the bag. The magnitude of the resultant was 24 lb.

(c) (i) Sketch a diagram to show each force in component form and the resultant force.



(ii) Find the measurement of the angle between the resultant vector and the vector of the 10 lb force to the nearest degree.





By Cos. law,

$15^2 = 24^2 + 10^2 - 2(10)(24)\cos\theta$

$\theta = 20^\circ$ (To the nearest degree)

(iii) According to the above situation, the two forces from different directions are applied to the handbag, can the magnitude of the resultant be larger than the sum of the magnitudes of the forces? Justify your answer.

No, the sum of two sides of a triangle is greater than the third ; therefore, the magnitude of the resultant is less than the sum of the two forces.

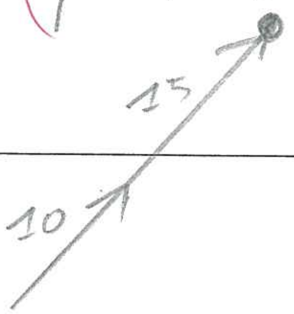
ie. $15 + 10 > 24$

(iv) What if the forces were from the same direction? How would it affect the magnitude of the resultant force? Does it make sense in real life? Briefly justify your answer.

The magnitude of the resultant is equal the sum of the two forces if the forces are from the same direction.

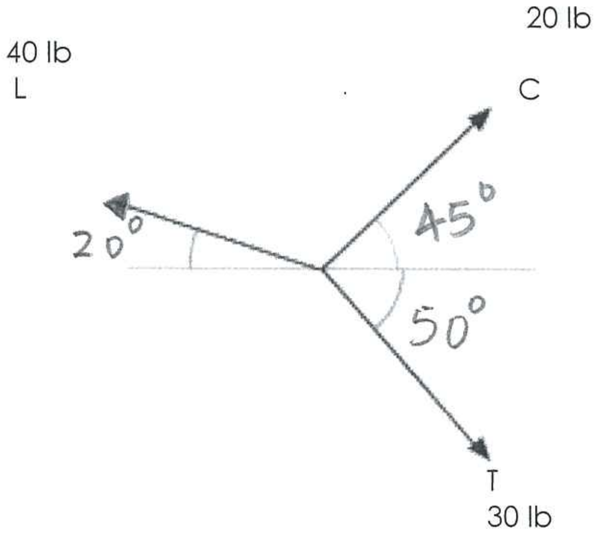
(Yes, it makes sense)

or no ∵ thief + cop won't be pulling from the same direction



$15 + 10 = 25$

Finally, L caught up with T and C, she joined the fight as well. The diagram shows the 3 forces.



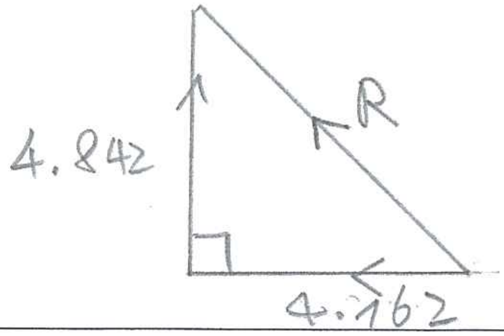
(d) The three forces shown in the diagram act at a point. Find the magnitude of their resultant and draw a diagram to show its directions. Who would have greater chance to get the handbag?

C 20 lb force $= \begin{pmatrix} 20 \cos 45° \\ 20 \sin 45° \end{pmatrix} \begin{matrix} = 14.14 \\ = 14.14 \end{matrix}$

L 40 lb force $= \begin{pmatrix} -40 \cos 20° \\ 40 \sin 20° \end{pmatrix} \begin{matrix} = -37.59 \\ = 13.68 \end{matrix}$

T 30 lb force $= \begin{pmatrix} 30 \cos 50° \\ -30 \sin 50° \end{pmatrix} \begin{matrix} = 19.28 \\ = -22.98 \end{matrix}$

Resultant force $= \begin{pmatrix} -4.162 \\ 4.842 \end{pmatrix}$



By pyth. Thm.,

$R^2 = 4.842^2 + 4.162^2$

$R \approx 6.38$

$\therefore$ L has greater chance to get the handbag.

(e) Explain how this answer makes sense in the context of this problem.

Yes, the lady was desperate to get her handbag. And, she was the last one to join the fight, she had more energy to get the bag.

No, the 2 men should be more powerful than a woman.. Therefore, the answer does not make sense.

End of Assessment